Letters

Comments on "On the Relationship Between TLM and Finite-Difference Methods for Maxwell's Equations"

WOJCIECH K. GWAREK

In the above paper,[1] Mr. Johns compares the finite-difference time-domain (FD–TD) and transmission-line matrix (TLM) methods and concludes that:

> In the three-dimensional TLM method operated in the above way, there are three field quantities available at each shunt and series node. This, for example, allows the boundary description for TLM to be twice as fine as for finite differences. In two dimensions, if boundaries are described only at nodes as in finite differences, the incident pulses need only be at alternate nodes at any instant. Thus, an average of two stores for link lines, not four, is required at each node.

The above statements are misleading since they suggest that in the FD–TD algorithm the boundary must coincide with the nodes. In fact, the parameters of the meshes situated at the boundary can be modified to simulate its particular shape. Such a boundary matching procedure for the general three-dimensional case is difficult to introduce and has not been reported yet, but for two dimensions it was introduced in [1]. My experience shows that, using the procedure described in [1], it is possible to compare two 2-D circuits differing in size by less than 0.1 of the mesh size (which is clearly not possible when using a TLM algorithm).

In general, the TLM method has a fundamental restriction on possible boundary shapes, requiring that the boundary pass through the nodes or through the points in the middle between them. Such a restriction is necessary to synchronize the incident and reflected pulses. The FD–TD algorithm does not have such a basic restriction, allowing a wide range of boundary matching procedures (although they may be difficult to implement).

The above remarks, critical to some of Mr. Johns's statements, do not undermine but rather support his final conclusion that "the TLM method and the finite difference method complement each other rather than compete with each other. Each leads to better understanding of the other."

Reply[2] *by Peter B. Johns*[3]

I would like to thank Dr. Gwarek for his interest and comments on my paper and I am pleased to have the opportunity of attempting to clarify points that have been misleading.

In my paper, I tried to show that the solution of certain transmission-line network models for electromagnetic fields could be expressed in terms of differences of the nodal or total field quantities only. Thus under particular conditions it is possible to eliminate the incident and reflected field quantities and obtain a conventional difference equation which gives numerically exactly the same numbers as a TLM routine. It was under these conditions, which are restricted, of course, that I was attempting to make detailed comparisons of the boundary description.

Dr. Gwarek, I think, is widening the comparison to areas where the methods may not result in exactly the same numbers at the network nodes or indeed where the network graphs may not be the same. Much greater care needs to be taken in making comparisons under these conditions, and certainly the advantages of different models depend very much on the application.

The description of boundaries in methods like finite differences has always been of interest and unequal arms were described a long time ago ([2], for example). A similar procedure for TLM where boundaries were modeled at an infinitely variable distance in the discretization mesh was described very early in the development of the method [3]. Variable meshes [4], which allow considerable flexibility in boundary placement, have been in use for a long time in two and three dimensions, and the modeling of boundaries at shallow angles to a general orthogonal mesh has also been briefly discussed [5]. Unfortunately the theory for determining whether the TLM method corresponds exactly to a finite-difference method for these types of modeling techniques has not been fully developed.

In the paper referred to by Dr. Gwarek [1], he draws the analogy between electromagnetic field equations and the equations of a lumped network. In his paper he describes, in effect, how the network equivalent for a regular square grid discretization can be modified to take account of the boundary shape. In Dr. Gwarek's comment, some confusion may be arising between the "nodes" of the original square mesh used for discretization and the nodes of the graph of the network modeling the boundary. As indicated above, the parameters in TLM may also be modified to simulate a boundary shape in an infinitely variable way. Both the TLM procedure and the finite difference procedure, however, are operated in terms of the node and branch variables of their network graphs.

TLM can be applied to the solution of quite arbitrary lumped networks [6]; it should be pointed out, therefore that Dr. Gwarek's boundary model lumped network can be solved in the time domain by TLM using link and stub transmission lines. For his boundary network model, it is likely that the finite-difference routine described by Dr. Gwarek gives the voltages at the network nodes and the currents in the branches half way between the network nodes only. The TLM routine would give the voltage and all branch currents at the nodes of the same network graph and the voltage and current on the branches half way between. This means that scattering could be introduced half way between the nodes simply by altering the scattering matrix at the nodes. However, the numbers obtained by the TLM method may not be exactly the same as the numbers obtained by the finite-difference routine because the restrictive condition on the global scattering matrix given in my paper may not apply.

However, I would like to emphasize most strongly that while comparisons in computer resources between finite differences and TLM (under the restricted conditions mentioned) may be interesting, the important difference in the methods, in my opin-

Manuscript received April 13, 1987.
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IEEE Log Number 8715974

[1] P. B. Johns, *IEEE Trans. Microwave Theory Tech*, vol. MTT-35, pp. 60–61, Jan. 1987

[2] Manuscript received May 20, 1987
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IEEE Log Number 8715975

ion, is the difference in modeling philosophy. Some engineers prefer to think of time-domain discretization through mathematical finite differencing; others prefer to model with transmission-line networks. Comfort in the modeling concept is far more likely to lead the modeler to more advanced models, as is illustrated by Dr. Gwarek in his interesting paper [1].

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Indefinite Integrals Useful in the Analysis of Cylindrical Dielectric Resonators

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Abstract —**Little-known integrals are listed, useful for the evaluation of stored electric energy in cylindrical regions, such as often appear in the analysis of cylindrical dielectric resonators.**

In the analysis of shielded dielectric resonators, it is often necessary to evaluate the stored electric or magnetic energy within a cylindrical region, such as regions 1, 2, and 3 in Fig. 1. The components of the electric field in region 1 are typically expressed in terms of the function

$$\phi_m(k\rho) = K_m(k\rho) + \alpha I_m(k\rho) \tag{1}$$

where $K_m(k\rho)$ and $I_m(k\rho)$ are the modified Bessel functions of order m, k is the radial wavenumber for the corresponding region, and α is a constant such that the tangential electric field vanishes at $\rho = b$. The boundary conditions at $z = 0$ and $z = L$ are not important in the present consideration. Either of these two surfaces may be covered with a perfect electric conductor or, alternatively, form an interface with a neighboring dielectric region.

When the stored electric energy in region 1 is required, the following indefinite integral is needed:

$$\int \left[\phi_m'^2(k\rho) + \frac{m^2}{k^2\rho^2}\phi_m^2(k\rho) \right] \rho\, d\rho = W(\rho). \tag{2}$$

The solution $W(\rho)$ cannot be found in common mathematical handbooks [1], [2]. Nevertheless, the solution exists as follows:

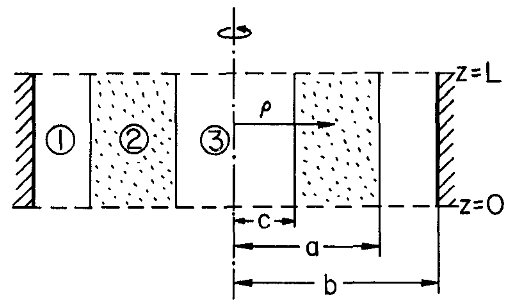

Fig. 1. Cylindrical region filled with inhomogeneous dielectric materials.

$$W(\rho) = \frac{\rho^2}{2}\left[\phi_m'^2(k\rho) + \frac{2}{k\rho}\phi_m(k\rho)\phi_m'(k\rho) - \left(1 + \frac{m^2}{k^2\rho^2}\right)\phi_m^2(k\rho) \right]. \tag{3}$$

When $\alpha = 0$, the result reduces to

$$\int \left[K_m'^2(k\rho) + \frac{m^2}{k^2\rho^2}K_m^2(k\rho) \right] \rho\, d\rho = \frac{\rho^2}{2}\left[K_m'^2(k\rho) + \frac{2}{k\rho}K_m(k\rho)K_m'(k\rho) - \left(1 + \frac{m^2}{k^2\rho^2}\right)K_m^2(k\rho) \right]. \tag{4}$$

The last formula can be found in [3], unfortunately distorted by typographical errors. This formula is useful when radius b of the cylindrical enclosure becomes infinitely large.

The proof of the above formulas consists of taking the derivative of the right-hand side of (3), and showing that

$$\frac{dW(\rho)}{d\rho} = \rho\left[\phi_m'^2(k\rho) + \frac{m^2}{k^2\rho^2}\phi_m^2(k\rho) \right]. \tag{5}$$

The derivation of the above identity is based on the fact that $\Phi_m''(k\rho)$, being a linear combination of modified Bessel functions, satisfies

$$\phi_m''(k\rho) = -\frac{1}{k\rho}\phi_m'(k\rho) + \left(1 + \frac{m^2}{k^2\rho^2}\right)\phi_m(k\rho). \tag{6}$$

Another, similar identity can be obtained for ordinary Bessel functions, needed for evaluation of the stored energy in region 2:

$$\int \left[\psi_m'^2(k\rho) + \frac{m^2}{k^2\rho^2}\psi_m^2(k\rho) \right] \rho\, d\rho = V(\rho) \tag{7}$$

where $\psi_m(k\rho)$ is a linear combination of the ordinary Bessel functions:

$$\psi_m(k\rho) = J_m(k\rho) + \beta Y_m(k\rho). \tag{8}$$

The corresponding solution is

$$V(\rho) = \frac{\rho^2}{2}\left[\psi_m'^2(k\rho) + \frac{2}{k\rho}\psi_m(k\rho)\psi_m'(k\rho) + \left(1 - \frac{m^2}{k^2\rho^2}\right)\psi_m^2(k\rho) \right]. \tag{9}$$

Manuscript received February 23, 1987. This material is based upon work supported by the National Science Foundation under Grant ECS-8443558.

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IEEE Log Number 8715977.

0018-9480/87/0900-0873$01.00 ©1987 IEEE